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# ON A REMARKABLE PROPERTY BELONGING TO SOME CUBICS.

BY C. O. BOIJE AF GENNAS, GOTHENBURG, SWEDEN.

THE well known theorem, Eucl. III, 31, can be enunciated as follows:

If through a point on the circumference of a circle two chords be drawn, making with each other a right angle, the straight line joining the extremities of the chords will pass through a fixed point, the centre of the circle.

Can a Cubic be found having the same, or analogous property? This question will be partially answered in the following discussion.

A straight line drawn through a point, or a cubic, will generally meet the curve in two other points. Let the investigation therefore be limited to that class of *symmetrical* cubics which are represented by the equation

$$y^2 = x^2 \frac{Ax+B}{Cx+D}, \quad (1)$$

the point through which the chords are to be drawn being the origin.

The equation of a straight line passing through the origin, and making an angle  $\theta$  with the axis of  $x$  is

$$y = x \tan \theta.$$

The coordinates of the point of intersection between (1) and (2) are

$$\left. \begin{aligned} x_1 &= \frac{D \tan^2 \theta - B}{A - C \tan^2 \theta}, \\ y_1 &= x_1 \tan \theta. \end{aligned} \right\} \quad (3)$$

Let  $\alpha$  be the angle included between the two chords, the coordinates of the point of intersection between the other end and the cubic are

$$\left. \begin{aligned} x_2 &= \frac{D \tan^2(\alpha - \theta) - B}{A - C \tan^2(\alpha - \theta)}, \\ y_2 &= -x_2 \tan(\alpha - \theta). \end{aligned} \right\} \quad (4)$$

The equation of the straight line passing through (3) and (4) is

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2} (x - x_1). \quad (5)$$

If this line shall pass through a fixed point, that point must be on the axis of  $x$  because of the symmetrical form of the cubic. Then, putting  $y = 0$  in the equation (5), we find the intersection between that line and the axis of  $x$  to be given by the abscissa

$$x = \frac{x_1 y_2 - x_2 y_1}{y_2 - y_1} = \frac{x_1 x_2 [\tan \theta + \tan(\alpha - \theta)]}{x_1 \tan \theta + x_2 \tan(\alpha - \theta)}, \quad (6), =$$

$$\frac{[D \tan^2 \theta - B][D \tan^2(\alpha - \theta) - B]}{AD[\tan^2 \theta - \tan \theta \tan(\alpha - \theta) + \tan^2(\alpha - \theta)] - CD \tan^2 \theta \tan^2(\alpha - \theta) + BC \tan \theta \tan(\alpha - \theta) - AB}$$

In order that this value may be independent of  $\theta$ ,  $\frac{dx}{d\theta}$  must be equal to zero. Substituting

$$\tan \theta \tan (\alpha - \theta) = t, \quad (7)$$

we get

$$\frac{dx}{d\theta} = \frac{[t^2(3D^2 + 2D^2 \tan^2 \alpha - BD \tan^2 \alpha) + (2Dt - B)(B - D \tan^2 \alpha)](BC - AD)}{[t^2(AD \tan^2 \alpha - CD) + t(BC - 2AD \tan^2 \alpha - 3AD) + (AD \tan^2 \alpha - AB)]^2} \cdot \frac{dt}{d\theta}, \quad \dots (8)$$

where the value of  $dt \div d\theta$  is given by the equation

$$\frac{dt}{d\theta} = \frac{\sin(2\alpha - 2\theta) - \sin 2\theta}{2 \cos^2 \theta \cos^2(\alpha - \theta)}. \quad (9)$$

Rejecting the solutions which will transform (1) into the equation of a conic, we see that  $dx \div d\theta$  can be made equal to zero, first, if

$$t^2(3D^2 + 2D^2 \tan^2 \alpha - BD \tan^2 \alpha) + (2Dt - B)(B - D \tan^2 \alpha) = 0, \quad (10)$$

which implies that the following eq's must be simultaneously satisfied ;

$$\left. \begin{aligned} 3D^2 + 2D^2 \tan^2 \alpha - BD \tan^2 \alpha &= 0 \\ D(B - D \tan^2 \alpha) &= 0 \\ B(B - D \tan^2 \alpha) &= 0 \end{aligned} \right\} \quad (11)$$

The only acceptable solution of these equations is

$$\left. \begin{aligned} B &= 3D, \\ \alpha &= 60^\circ (120^\circ). \end{aligned} \right\} \quad (12)$$

Then the equation of the cubic will be

$$y^2 = x^2 \frac{Ax + 3D}{Cx + D}, \quad (A')$$

and having drawn two chords through the origin, making with each other an angle of  $60^\circ (120^\circ)$ , the straight line joining the extremities of the ch'ds will pass through a fixed point on the axis of  $x$  situated at a distance from the origin, which from equation (6) we find to be

$$x = \frac{8D}{C - 3A}. \quad (B')$$

Secondly,  $dx \div d\theta$  can be made equal to zero, if

$$\frac{dt}{d\theta} = 0. \quad (13)$$

Hence it follows that

$$\alpha = 90^\circ. \quad (14)$$

From (6) we then deduce

$$x = \frac{B^2 - BD(\tan^2 \theta + \cot^2 \theta) + D^2}{AD(\tan^2 \theta + \cot^2 \theta - 1) - CD - AD + BC} \quad (15)$$

and

$$\frac{dx}{d\theta} = \frac{D(AD - BC)(D - B)(\sec^2 \theta - \operatorname{cosec}^2 \theta)}{[AD(\tan^2 \theta + \cot^2 \theta - 1) - CD - AB + BC]^2}. \quad (16)$$

Rejecting as in the former case solutions which will transform (1) into a conic, we see that  $dx \div d\theta$  is equal to zero if

$$D = B. \quad (17)$$

The eq'n of the cubic will then be

$$y^2 = x^2 \frac{Ax + B}{Cx + D}, \quad (A'')$$

and the straight line joining the extremities of two chords, drawn through the origin at right angles to each other, will pass through a fixed point on the axis of  $x$ , the abscissa of which, from eq'n (6) or (15) we find to be

$$x = -\frac{B}{A}. \quad (B'')$$

The fixed point can of course lie at an infinite distance from the origin; the right line joining the extremities of the chords will then be parallel to the axis of  $x$ .

Concerning the conics, where the constant angle is  $90^\circ$  and the point from which the chords are to be drawn can be taken anywhere on the curve, the investigation is of such a special interest that it deserves a separate treatise, which we hope to give in another number of this periodical.

## INTEGRATION OF THE GENERAL EQUATION OF MOTION.

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THE first difficulty which the student encounters in reading Gauss's *Theoria Motus Corporum Celestium*, is found in Article 3, where

$$ax + \beta y + r = \gamma$$

is given as the general equation of the conic sections.

The equation is, I believe, due to La Place, who gave a demonstration of it in the *Mécanique Céleste*, Book II, Chap. III; and therefore, for want of a better name, I shall take the liberty of calling it La Place's Equation to the Conic Sections. I purpose in this brief paper to give a short and easy demonstration of this equation and to discuss it with the view of ascertaining the significance of the constants  $\alpha$ ,  $\beta$ ,  $\gamma$ .

Let us assume the general equations of motion which are

$$\frac{d^2x}{dt^2} = -F \frac{x}{r} \dots (1), \text{ and } \frac{d^2y}{dt^2} = -F \frac{y}{r} \dots (2),$$

where  $F$  is the force. If  $F$  vary as  $1 \div r^2$ , as is the case in nature, then  $F = \mu \div r^2$ , where  $\mu$  is the unit of force at the unit of distance or the absolute force, and (1) and (2) become